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CP violation in the charged pion energy spectra of the decays $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

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Abstract

CP violation leads to a difference between the parameters g^+ and g^- that characterise the energy distributions of the "odd" pion in the decays $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ and $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$. We argue that for the first decay, the asymmetry $\Delta g = (g^+ - g^-)/(g^+ + g^-)$ is fixed at a value around $\Delta g = 2 \times 10^{-6}$, whereas for the second decay, the asymmetry Δg may be one order of magnitude larger.

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It is well known that the strength of direct CP violation in the $K_L \to 2\pi$ decays, as determined by the parameter ε', is crucially depending on the fact that the QCD penguin (QCDP) and the electroweak penguin (EWP) contributions partially cancel one another [1]. Thus, it is not difficult to understand that before the experimental value $\varepsilon'/\varepsilon = (1.67 \pm 0.26) \times 10^{-3}$ [2] was available the theoretical predictions for ε'/ε were one order of magnitude smaller than this value [3], or very uncertain, leading to values of this ratio varying all over the range $10^{-4} \leqslant \varepsilon'/\varepsilon \leqslant 10^{-3}$ [4,5]. In the present note we discuss some consequences for the $K^{\pm} \to 3\pi$ decays.

In [6–8], it was found that contrary to the case of ε', in $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ decay, the EWP contribution enhance the QCDP contribution. But in order to estimate the magnitude of the CP-violating effect, it was necessary to resort to unreliable theoretical estimates of the QCDP and the EWP contributions (see Ref. [8]).

For the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ decay, the situation is cleaner, because as explained in the present note, the CP-odd asymmetry Δg in this case turns out to be proportional to practically the same combination of QCDP and EWP contributions as in ε'. Consequently, Δg can be estimated reliably using the known value of ε'. For the $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$ decay, on the other hand, we argue that Δg may be one order of magnitude larger than in the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ decay. This conclusion differs from those proposed in Refs. [9,10].

Our investigation is based on the effective $\Delta S = 1$ non-leptonic Lagrangian proposed in Ref. [11],

$$\mathcal{L}(\Delta S = 1) = \sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_i c_i O_i, \tag{1}$$

where the O_i are four-quark operators, defined as

$$\begin{aligned}
O_1 &= \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L - \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L, \\
O_2 &= \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L \\
&\quad + 2\bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L + 2\bar{s}_L\gamma_\mu d_L \cdot \bar{s}_L\gamma_\mu s_L, \\
O_3 &= \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L \\
&\quad + 2\bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L - 3\bar{s}_L\gamma_\mu d_L \cdot \bar{s}_L\gamma_\mu s_L, \\
O_4 &= \bar{s}_L\gamma_\mu d_L \cdot \bar{u}_L\gamma_\mu u_L + \bar{s}_L\gamma_\mu u_L \cdot \bar{u}_L\gamma_\mu d_L \\
&\quad - \bar{s}_L\gamma_\mu d_L \cdot \bar{d}_L\gamma_\mu d_L,
\end{aligned}$$

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$$O_5 = \bar{s}_L \gamma_\mu \lambda^a d_L \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu \lambda^a q_R \right),$$

$$O_6 = \bar{s}_L \gamma_\mu d_L \left(\sum_{q=u,d,s} \bar{q}_R \gamma_\mu q_R \right). \tag{2}$$

For our study of CP violation, we must add two more four-quark operators,

$$O_7 = \frac{3}{2} \bar{s} \gamma_\mu (1+\gamma_5) d \left(\sum_{q=u,d,s} e_q \bar{q} \gamma_\mu (1-\gamma_5) q \right),$$

$$O_8 = -12 \sum_{q=u,d,s} e_q (\bar{s}_L q_R)(\bar{q}_R d_L), \tag{3}$$

where e_q is the quark-charge matrix.

The operators $O_{5,6}$ arise from the QCD penguin diagram and the operators $O_{7,8}$ arise, analogously, from the electroweak penguin diagram. The Wilson coefficients $c_{5–8}$ contain the imaginary parts necessary for CP violation. The bosonization of the operators $O_{1–8}$ can be achieved by exploiting the relations between di-quark field operators and pseudoscalar fields as represented in [12], and the reordering relations in colour and spinor spaces as from [13].

Representing the $K \to 2\pi$ amplitudes in the form

$$M\left(K_1^0 \to \pi^+\pi^-\right) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2},$$

$$M\left(K_1^0 \to \pi^0\pi^0\right) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2}, \tag{4}$$

this approach yields

$$A_0 = \kappa \left[c_1 - c_2 - c_3 + \frac{32}{9} \beta (\operatorname{Re} \tilde{c}_5 + i \operatorname{Im} \tilde{c}_5) \right], \tag{5}$$

$$A_2 = \kappa \left[c_4 + i \frac{2}{3} \beta \Lambda^2 \operatorname{Im} \tilde{c}_7 \left(m_K^2 - m_\pi^2\right)^{-1} \right]. \tag{6}$$

Here, δ_0 and δ_2 are the pion–pion scattering phase shifts in the isospin $T = 0$ and $T = 2$ channels, and the remaining parameters are

$$\kappa = G_F F_\pi \sin\theta_C \cos\theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}}, \qquad \beta = \frac{2m_\pi^4}{\Lambda^2 (m_u + m_d)^2},$$

$$\tilde{c}_5 = c_5 + \frac{3}{16} c_6, \qquad \tilde{c}_7 = c_7 + 3c_8, \qquad \Lambda \approx 1 \text{ GeV}.$$

Since $\tilde{c}_7/\tilde{c}_5 \sim \alpha_{\text{em}}$ and small, we have neglected the EWP contribution to A_0.

From data on $K \to 2\pi$ rates one can deduce the values of the real parts of the amplitudes A_0 and A_2 [14], i.e.,

$$c_4 = 0.328, \tag{7}$$

$$c_1 - c_2 - c_3 + \frac{32}{9} \beta \operatorname{Re} \tilde{c}_5 = -10.13. \tag{8}$$

Furthermore, if as suggested by Refs. [11,13], we assume $c_1 - c_2 - c_3 = -2.89$, then we have in addition $\frac{32}{9} \beta \operatorname{Re} \tilde{c}_5 = -7.24$.

Using the definition of the parameter ε',

$$\varepsilon' = i e^{i(\delta_2 - \delta_0)} \left[-\frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} + \frac{\operatorname{Im} A_2}{\operatorname{Re} A_2} \right] \cdot \left| \frac{A_2}{A_0} \right|, \tag{9}$$

and its experimental value, we deduce

$$-\frac{\operatorname{Im} \tilde{c}_5}{\operatorname{Re} \tilde{c}_5} \left(1 - \Omega + 24.4 \frac{\operatorname{Im} \tilde{c}_7}{\operatorname{Im} \tilde{c}_5} \right) = (1.63 \pm 0.16) \times 10^{-4}. \tag{10}$$

The new parameter Ω takes into account effects of isospin violation, coming from the quark mass difference $m_d \neq m_u$ and the electromagnetic interaction. As a result hereof, the physical state vector of the isovector $I = 1$ neutral pi-meson acquires an admixture of states with isospin $I = 0$,

$$\left|\pi^0_{\text{phys}}\right\rangle = \left|\pi^0\right\rangle + \lambda|\eta\rangle + \lambda'|\eta'\rangle. \tag{11}$$

For a recent review see Ref. [15].

As a consequence of mixing there are alternative contributions to the $K \to \pi^0\pi^0$ decays. The weak-interaction Lagrangian of Eq. (1) can in a first step induce the transitions $K^0 \to \pi^0\eta(\eta')$ which, in a second step, are followed by the transitions $\eta(\eta') \to \pi^0$ induced by the isospin mixing of Eq. (11). Thus, in the tree approximation, the isospin decompositions of the $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$ amplitudes change into

$$\begin{aligned} \left\langle \pi^+\pi^- \middle| \mathcal{H}_w \middle| K^0 \right\rangle &= (A_0 - \gamma)^{(I=0)} - (A_2 - \gamma)^{(I=2)} \\ &= A_0 - A_2, \\ \left\langle \pi^0\pi^0 \middle| \mathcal{H}_w \middle| K^0 \right\rangle &= (A_0 - \gamma)^{(I=0)} + 2(A_2 - \gamma)^{(I=2)} \\ &= A_0 + 2A_2 - 3\gamma. \end{aligned}$$

Here, $-\gamma$ is the matrix element coming from the mixing. It is the same for both isospin channel amplitudes and 1/3 of the amplitude for the $K^0 \to \eta(\eta') \to \pi^0\pi^0$ transition.

Now, in the absence of EWP contributions, the combination

$$-\frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} + \frac{\operatorname{Im} A_2}{\operatorname{Re} A_2} \tag{12}$$

of Eq. (9) transforms into

$$\begin{aligned} &-\frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} \left[1 - \frac{\operatorname{Im} \gamma}{\operatorname{Im} A_0} + \frac{\operatorname{Re} A_0}{\operatorname{Re} A_2} \frac{\operatorname{Im} \gamma}{\operatorname{Im} A_0} \left(1 + \frac{\operatorname{Re} \gamma}{\operatorname{Re} A_2} \right) \right] \\ &\quad \equiv -\frac{\operatorname{Im} A_0}{\operatorname{Re} A_0} [1 - \Omega], \end{aligned} \tag{13}$$

where only the most important terms have been retained. Calculations [16] of Ω to leading order in a low-energy expansion suggested $\Omega = 0.25 \pm 0.10$ [17]. However, it was later found that the next-to-leading order corrections reduce the value of Ω to $\Omega = 0.16 \pm 0.03$ [18], and possibly to an even to smaller value, $\Omega_{\text{eff}} = 0.060 \pm 0.077$, providing electromagnetic corrections (without EWP though) are considered as well [19]. Taking this last value of Ω_{eff} as our preferred one we can rewrite Eq. (10) as

$$\beta \operatorname{Im} \tilde{c}_5 \left[1 + (26.1 \pm 2.1) \frac{\operatorname{Im} \tilde{c}_7}{\operatorname{Im} \tilde{c}_5} \right] = (3.56 \pm 0.61) \times 10^{-4}. \tag{14}$$

Below, we shall see that the numerical result of Eq. (14) leads to a reliable estimate of the asymmetry parameter Δg in the $K^\pm \to \pi^0\pi^0\pi^\pm$ decay.

Let us now turn to the $K^\pm \to 3\pi$ decays. Applying the same techniques as above and taking into account the appearance of

CP-even imaginary parts due the to strong $\pi\pi$ final-state rescattering, we get in leading p^2 approximation for the τ and τ' decay amplitudes:

$$M\big(K^{\pm}(k)\to\pi^{\pm}(p_1)\pi^{\pm}(p_2)\pi^{\mp}(p_3)\big)$$
$$=\tilde{\kappa}\left[1+ia+\frac{1}{2}g_\tau Y\big(1+ib^\tau\pm id^\tau_{\mathrm{KM}}\big)+\cdots\right], \tag{15}$$
$$M\big(K^{\pm}(k)\to\pi^{0}(p_1)\pi^{0}(p_2)\pi^{\pm}(p_3)\big)$$
$$=\frac{\tilde{\kappa}}{2}\left[1+ia+\frac{1}{2}g_{\tau'} Y\big(1+ib^{\tau'}\pm id^{\tau'}_{\mathrm{KM}}\big)+\cdots\right]. \tag{16}$$

The indices τ and τ' refer to the decay modes of the kaon. The parameters a, b^τ and $b^{\tau'}$ arise from the strong pion–pion rescattering and are consequently CP-even. The $d^{\tau,\tau'}_{\mathrm{KM}}$ are CP-odd imaginary terms produced by the Kobayashi–Maskawa phase. Furthermore, Y is a kinematic factor, $Y=(s_3-s_0)/m_\pi^2$, with $s_3=(k-p_3)^2$ and $s_0=\frac{1}{3}m_K^2+m_\pi^2$.

In $K^\pm\to\pi^\pm\pi^\pm\pi^\mp$ decay, the parameter values are

$$a=0.12,\qquad b^\tau=0.71,\qquad g_\tau=-\frac{3m_\pi^2}{m_K^2}(1+9c_4/c_0),$$

$$c_0=c_1-c_2-c_3-c_4+\frac{32}{9}\beta\,\mathrm{Re}\,\tilde{c}_5=-10.46,$$
$$\tilde{\kappa}=G_F m_K^2\sin\theta_C\cos\theta_C c_0/3\sqrt{2},$$

and for the CP-odd contribution we get

$$d^\tau_{\mathrm{KM}}=-\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5\frac{9c_4}{c_0(c_0+9c_4)}$$
$$\times\left[1+\frac{3\Lambda^2(c_0+9c_4)}{16m_K^2c_4}\left(1+\frac{12c_4m_K^2}{\Lambda^2(c_0+9c_4)}\right)\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right]$$
$$=-2\frac{16c_4}{c_0(c_0+9c_4)}\beta\,\mathrm{Im}\,\tilde{c}_5\left(1-14.36\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right). \tag{17}$$

In $K^\pm\to\pi^0\pi^0\pi^\pm$ decay, two parameters are different, i.e.,

$$b^{\tau'}=0.49,\qquad g_{\tau'}=\frac{6m_\pi^2}{m_K^2}(1-9c_4/2c_0),$$

as is the CP-odd contribution

$$d^{\tau'}_{\mathrm{KM}}=\frac{32}{9}\beta\,\mathrm{Im}\,\tilde{c}_5\frac{9c_4/2}{c_0(c_0-9c_4/2)}$$
$$\times\left[1-\frac{3\Lambda^2(c_0-9c_4/2)}{8m_K^2c_4}\right.$$
$$\left.\times\left(1-\frac{c_0m_K^2}{2(c_0-9c_4/2)(m_K^2-m_\pi^2)}\right)\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right]$$
$$=\frac{16c_4}{c_0(c_0-9c_4/2)}\beta\,\mathrm{Im}\,\tilde{c}_5\left(1+27.8\frac{\mathrm{Im}\,\tilde{c}_7}{\mathrm{Im}\,\tilde{c}_5}\right). \tag{18}$$

The slope parameters $g^\pm_\tau$ in τ decay are defined by the equation

$$\big|M\big(K^{\pm}(k)\to\pi^{\pm}(p_1)\pi^{\pm}(p_2)\pi^{\mp}(p_3)\big)\big|^2\sim\big[1+g^\pm_\tau Y+\cdots\big] \tag{19}$$

with a similar definition for $g^\pm_{\tau'}$ in τ' decay. The CP-asymmetry parameters $\Delta g_{\tau,\tau'}$ in the two decays are defined as

$$\Delta g_\tau=\frac{g^+_\tau-g^-_\tau}{g^+_\tau+g^-_\tau}=\frac{ad^\tau_{\mathrm{KM}}}{1+ab^\tau},$$
$$\Delta g_{\tau'}=\frac{g^+_{\tau'}-g^-_{\tau'}}{g^+_{\tau'}+g^-_{\tau'}}=\frac{ad^{\tau'}_{\mathrm{KM}}}{1+ab^{\tau'}}. \tag{20}$$

Discussing first τ' decay, we realise when comparing Eqs. (18) and (14) that the linear combinations of QCDP and EWP contributions appearing in these expressions are very similar. In fact, at $\Omega=0.124$ the two combinations are identical. Thus, exploiting our knowledge of the experimental value of ε' we predict for the asymmetry parameter of Eq. (20)

$$\Delta g_{\tau'}=(1.8\pm0.24)\times10^{-6}. \tag{21}$$

At $\Omega_{\mathrm{eff}}=0.060\pm0.077$ from Ref. [19]

$$\Delta g_{\tau'}=(1.71\pm0.29)\times10^{-6}. \tag{22}$$

We conclude that due to the close resemblance of the expressions for ε' and $\Delta g_{\tau'}$ decay our prediction for $\Delta g_{\tau'}$ should be quite robust.

The CP-asymmetry parameter in τ decay is most easily discussed via the ratio,

$$\frac{-\Delta g_\tau}{\Delta g_{\tau'}}=2\left(\frac{c_0-9c_4/2}{c_0+9c_4}\right)\left(\frac{1+ab^{\tau'}}{1+ab^\tau}\right)$$
$$\times\left(\frac{1-14.36\,\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5}{1+27.8\,\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5}\right), \tag{23}$$

which is obtained by combining Eqs. (17)–(19). Therefore,

(a) if the EWP contributions do not play any significant role in direct CP violation, i.e., when $\mathrm{Im}\,\tilde{c}_7/\mathrm{Im}\,\tilde{c}_5$ is negligibly small, then

$$-\Delta g_\tau/\Delta g_{\tau'}=3.1\quad\text{or}\quad-\Delta g_\tau\geqslant0.56\times10^{-5}; \tag{24}$$

(b) if the EWP contribution cancels half of the QCDP contribution in ε' (see Refs. [20,21]), then

$$-\Delta g_\tau=7.8\Delta g_{\tau'}\geqslant1.3\times10^{-5}. \tag{25}$$

The above results are obtained in leading p^2 approximation. The role of p^4 corrections for Δg_τ were studied in Refs. [6,8], and they were found to increase the value of Δg_τ by 23%. For $\Delta g_{\tau'}$ the corresponding investigation has not yet been performed. But one effect can be seen at once. According to Refs. [6,8], the corrections of order p^4 increase the rescattering parameter a in Eq. (20) by 30%. Thus, we expect the corrected value of $\Delta g_{\tau'}$ to lie in the range (1.8–2.5) $\times10^{-6}$.

Finally, we remark once more that our numerical results not only differ from those reported in [9], which are $-\Delta g_\tau=(2.3\pm0.6)\times10^{-6}$ and $\Delta g_{\tau'}=(1.3\pm0.4)\times10^{-6}$, but also from the more recent ones reported in [10], which are $-\Delta g_\tau=(2.4\pm1.2)\times10^{-5}$ and $\Delta g_{\tau'}=(1.1\pm0.7)\times10^{-5}$. Both investigations were performed within the framework of chiral perturbation theory. In Ref. [10] attempts were made to estimate contributions of order p^4, but the predicted value for $\Delta g_{\tau'}$ has large uncertainties.

Our results strongly suggest, that accurate measurements of Δg_τ and $\Delta g_{\tau'}$ should clarify the relative importance of QCDP and EWP mechanisms in direct CP violation.

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